INTRODUCTION TO SIMULATION

Unit 1

What is Simulation??

- A Simulation is the imitation (artificial) of the operation of a real-world process or system over time. Whether done by ***hand*** or on a ***computer***, simulation involves the generation of an artificial history of a system and the observation of that artificial history to draw inferences concerning the operating characteristics of the real system.
- A simulation of a system is the operation of a model of the system.
- The model can be reconfigured and experimented with; usually, this is impossible, too expensive or impractical to do in the system it represents.

What is Simulation??

- Simulation is ***used before an existing system*** is altered or a ***new system built***, to reduce the chances of **failure** to meet specifications, to eliminate unforeseen bottlenecks, to prevent under or over-utilization of resources, and to optimize system performance.
- Simulation is the numerical technique for conducting experiments on digital computer, which involves logical and mathematical relationships that interact to describe the behavior and the structure of a complex real world system over extended period of time.

What is Simulation??

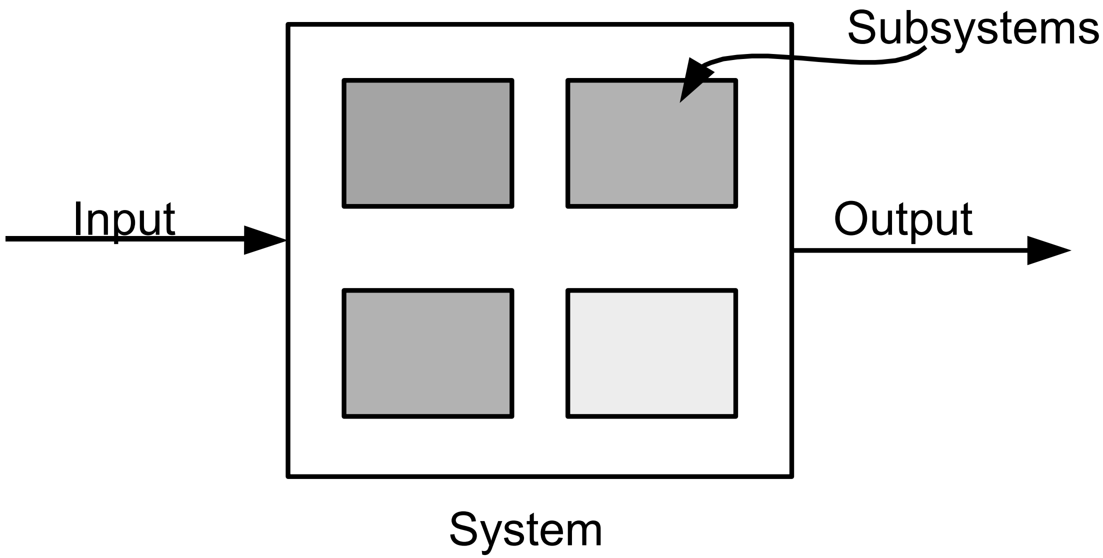
- The behavior of a system as it evolves over time is studied by developing a simulation model. This model usually takes the form of a set of assumptions concerning the operation of the system. These assumptions are expressed
  - mathematical,
  - logical, and
  - symbolic

System

- The term system is derive from the Greek word systema, which means an organized relationship among functioning units or components.
- A system is defined as a group of objects that are joined together in some regular interaction or interdependence for the accomplishment of some task. For example: *Production system for manufacturing automobiles.*
- A system is usually considered as a set of inter–related factors, which are described as entities activities and have properties or attributes.

System

- Assembly of objects joined in some regular interaction or interdependence.
- A system exists and operates in time and space.
- bounded inside system boundary



Example of System

1. **Aircraft flying under the control of an auto pilot.** The block diagram of autopilot aircraft is shown below:

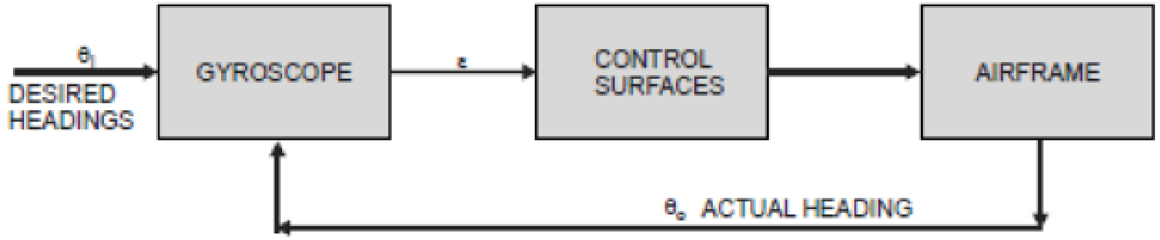


Fig: Model of an autopilot aircraft

A gyroscope in the auto pilot aircraft detects the difference between the actual heading and desired heading. It sends a signal, to move, to the control system. In response to control system the airframe steers towards the desired heading.

Example of System

2. Factory System

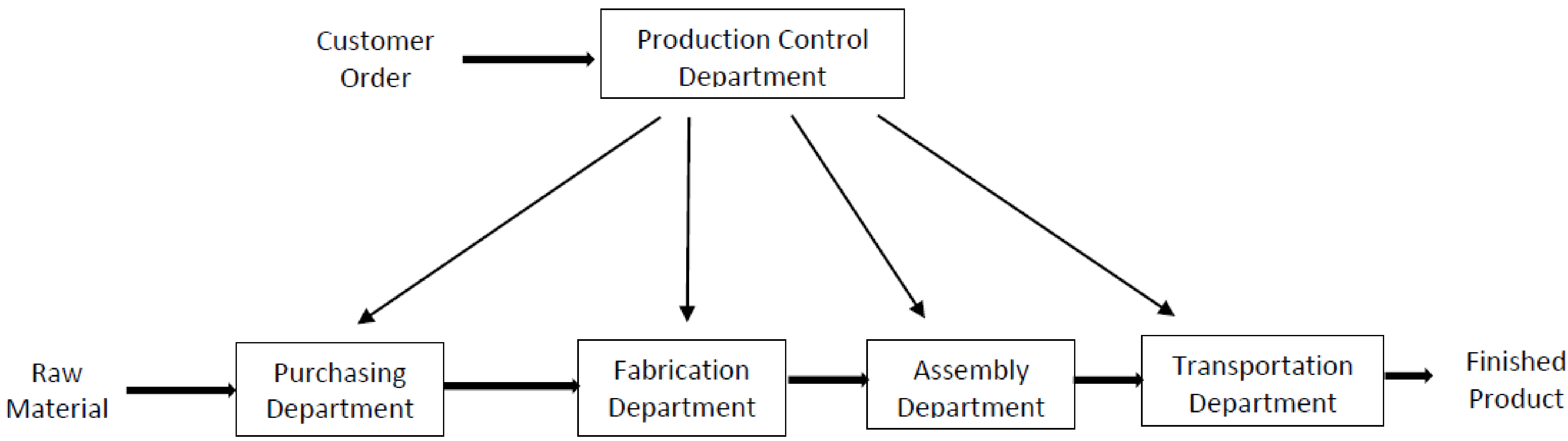


Fig: Factory System

- Consider a factory system that takes orders from customers and produces a finished product. In the above system, Production Control Department takes orders from customers and direct Purchasing Department to purchase raw material to carry out order. The Fabrication Department fabricates the raw material and the fabricated raw materials are assembled by Assembly Department and are sent to Shipping Department (TD) for the dispatch of finished product.

Components of a System:

- **Entity**
  - An entity is an object of interest in a system. Ex: In the factory system, departments, orders, parts and products are The entities.
- **Attribute**
  - An attribute denotes the property of an entity. Ex: Quantities for each order, type of part, or number of machines in a Department are attributes of factory system.
- **Activity**
  - Any process causing changes in a system is called as an activity. Ex: Manufacturing process of the department.
- **State of the System**
  - The state of a system is defined as the collection of variables necessary to describe a system at any time, relative to the objective of study. In other words, state of the system mean a description of all the entities, attributes and activities as they exist at one point in time.

Components of a System:

Event

- An event is define as an instaneous occurrence that may change the state of the system.

System	Entities	Attributes	Activities
Traffic System	bikes, cars, light, road	speed, turning angle, colors	driving, lightning, turning
Canteen System	manager, cooks, server, customer	manager_name, customer_order, menu	ordering, cooking, serving

Components of a System:

Table 1.1. Examples of Systems and Components

System	*Entities*	*Attributes*	*Activities*	*Events*	*State Variables*
Banking	Customers	Checking account balance	Making deposits	Arrival; departure	Number of busy tellers; number of customers waiting
Rapid rail	Riders	Origination; destination	Traveling	Arrival at station; arrival at destination	Number of riders waiting at each station; number of riders in transit
Production	Machines	Speed; capacity; breakdown rate	Welding; stamping	Breakdown	Status of machines (busy, idle, or down)
Communications	Messages	Length; destination	Transmitting	Arrival at destination	Number waiting to be transmitted
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

System Environment

- The external components which interact with the system and produce necessary changes are said to constitute the system environment.
- In modeling systems, it is necessary to decide on the boundary between the system and its environment. This decision may depend on the purpose of the study.
- Ex: In a factory system, the factors controlling arrival of orders may be considered to be outside the factory but yet a part of the system environment. When, we consider the demand and supply of goods, there is certainly a relationship between the factory output and arrival of orders. This relationship is considered as an activity of the system.

System Environment

The changes occurring outside the system are said to occur in system environment.

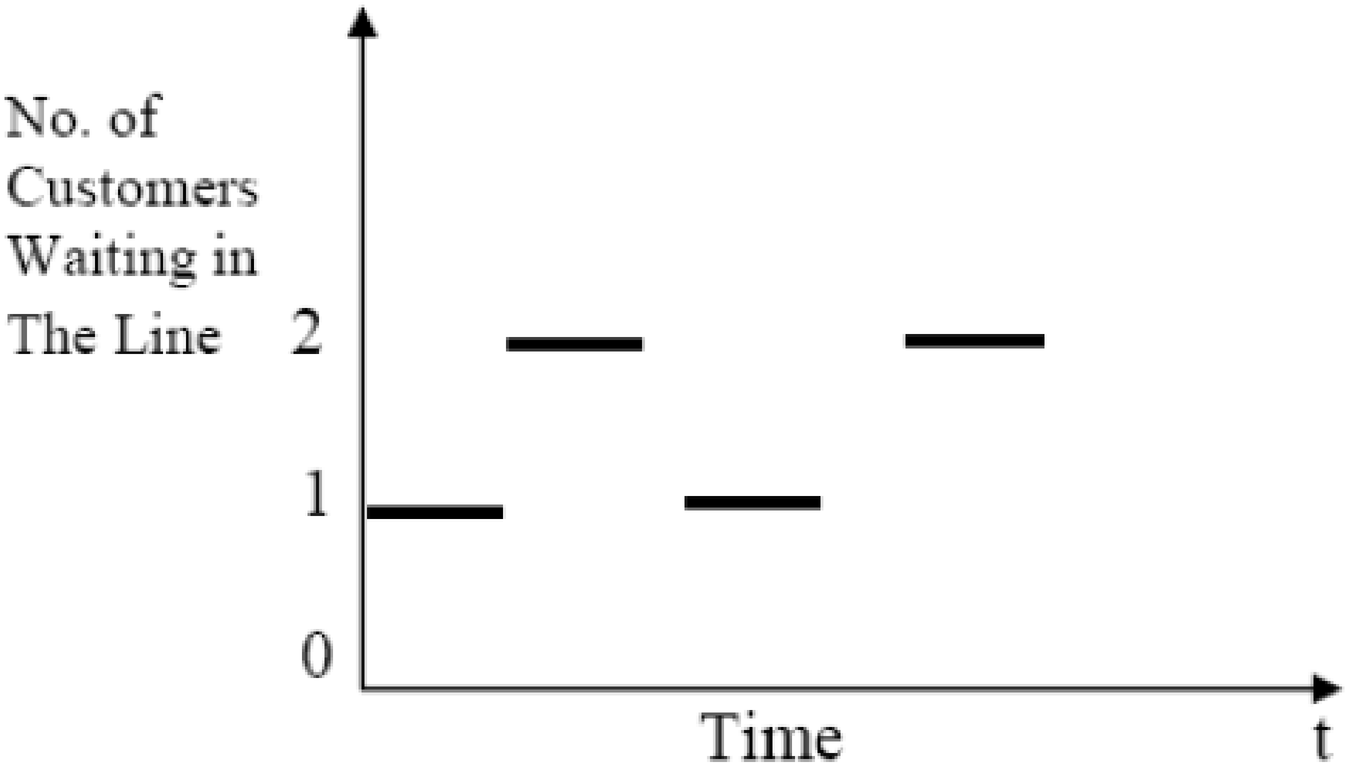
- **Endogenous System**
  - The term endogenous is used to describe activities and events occurring within a system. Ex: Drawing cash in a bank.
- **Exogenous System**
  - The term exogenous is used to describe activities and events in the environment that affect the system. Ex: Arrival of customers.
- **Closed System**
  - A system for which there is no exogenous (No External) activity and event is said to be a closed. Ex: Water in an insulated flask.
- **Open system**
  - A system for which there is exogenous(External) activity and event is said to be a open. Ex: Bank system.

Discrete System and Continuous System

Discrete Systems

- A discrete system is one in which the state variable(s) change only at a ***discrete set*** of points in time.
- The bank is an example of a discrete system: *The state variable, the number of customers in the bank, changes only when a customer arrives or when the service provided a customer is completed shows how the number of customers changes only at discrete points in time.*
- i.e. Systems in which the changes are predominantly (mostly) discontinuous are called discrete systems.
- So, A discrete system is one for which the state variables changes instantaneously (immediately) at separate point of time. E.g. Factory System, Bank System

Discrete Systems

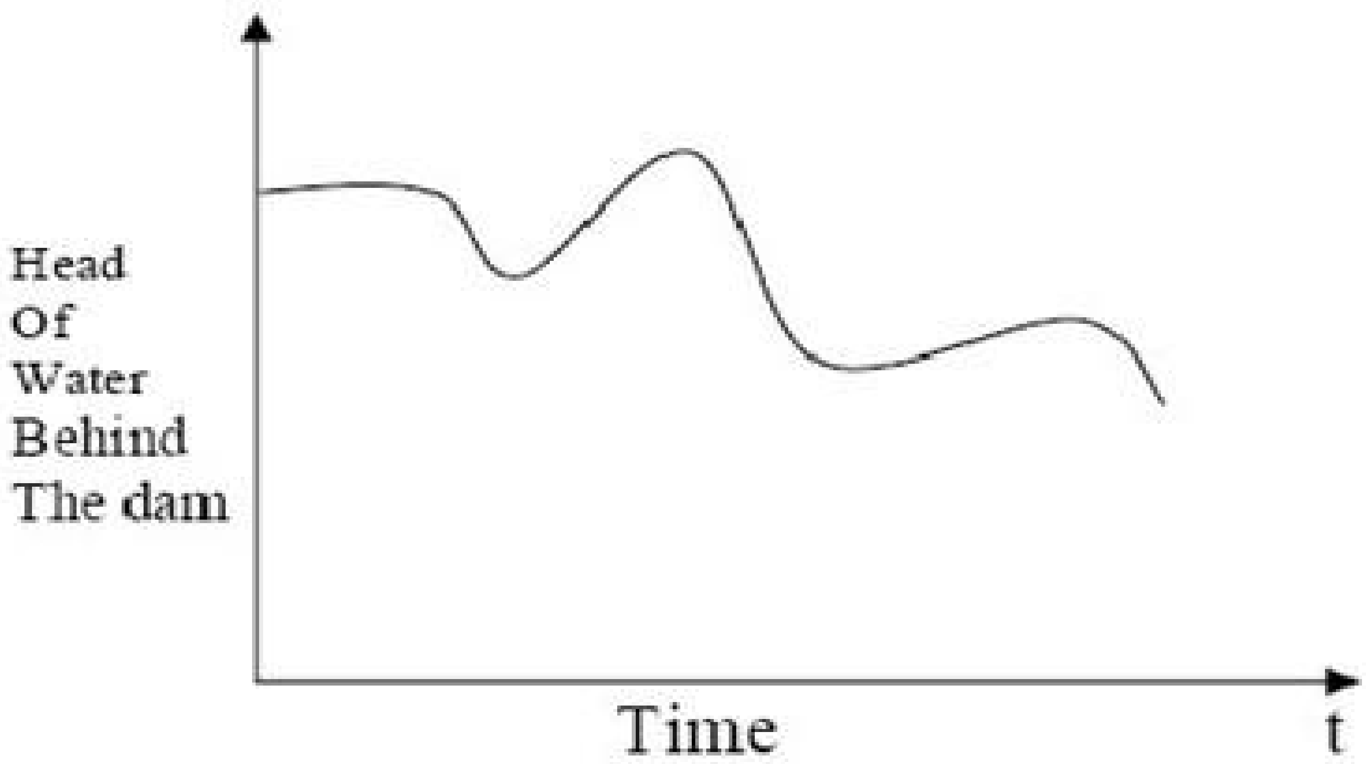


Discrete System and Continuous System

Continuous Systems

- A continuous system is one in which the state variable(s) change continuously over time.
- An example is the head of water behind a dam. During and for some time after a rain storm, water flows into the lake behind the dam. Water is drawn from the dam for flood control and to make electricity. Evaporation also decreases the water level. Figure shows how the state variable head of water behind the dam changes for this continuous system.
- i.e. A continuous system is one for which the state variables changes continuously with respect to time. E.g. Head of Water behind the dam

Continuous Systems



Real time simulation

- Real time simulation refers to a computer model of a physical system that can execute at the same rate as actual "wall clock" time.
- In other words, the computer model runs at the same rate as the actual physical system.
- For example if a tank takes 10 minutes to fill in the real-world, the simulation would take 10 minutes as well.
- Real-time simulation occurs commonly in computer gaming, statistical power grid protection tests, aircraft design and simulation, motor drive controller design methods and space robot integration are a few examples of real-time simulator technology applications.

Real time simulation

- Real time simulation is an approach where an actual device (hardware or software) can be used rather than constructing a model
- With this techniques actual devices which are part of a system are used in conjunction with either a digital or hybrid computer, providing a simulation of the parts of the system that do not exist or that can not continently used in an experiment
- Real time simulation will often involve interaction with a human being , there by avoiding the need to design and validate a model of human behavior

Real time simulation

- Real time simulation requires computers that receive signals and respond to it which are sent from physical devices and transfer output signals at specific points in time
- Example: Devices for training pilots by giving them the impression they are at the controls of an aircraft in such an environment where gravity is 1/6th part of earth and trained to perform different activities

Model of System

A model is defined as a representation of a system for the purpose of studying the system.

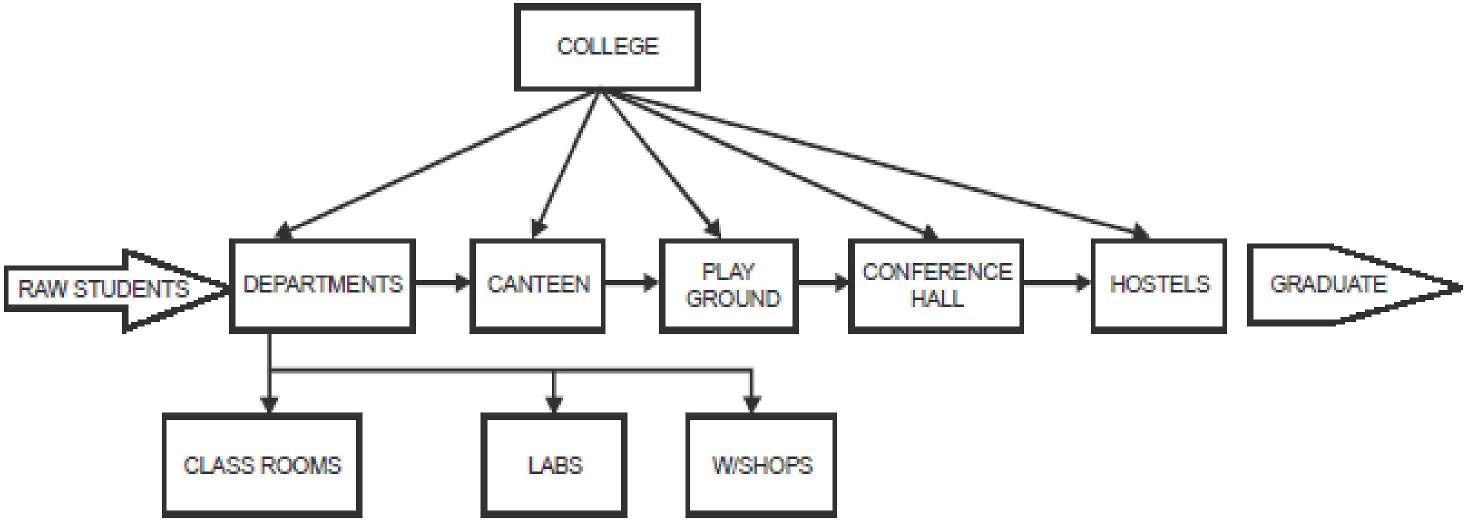
It is necessary to consider only those aspects of the system that affect the problem under investigation.

These aspects are represented in a model, and by definition it is a simplification of the system

Principles used in modeling:

1. Block building:

- The description of the system should be organized in a series of blocks to simplify the specifications of the interactions within the system.
- Each block represents a part of the system and the system as a whole can be described in terms of interconnections between blocks.



Principles used in modeling:

2. Relevance:

- The model should only include those aspects of the system that are **relevant** (***applicable***) to the study objectives.

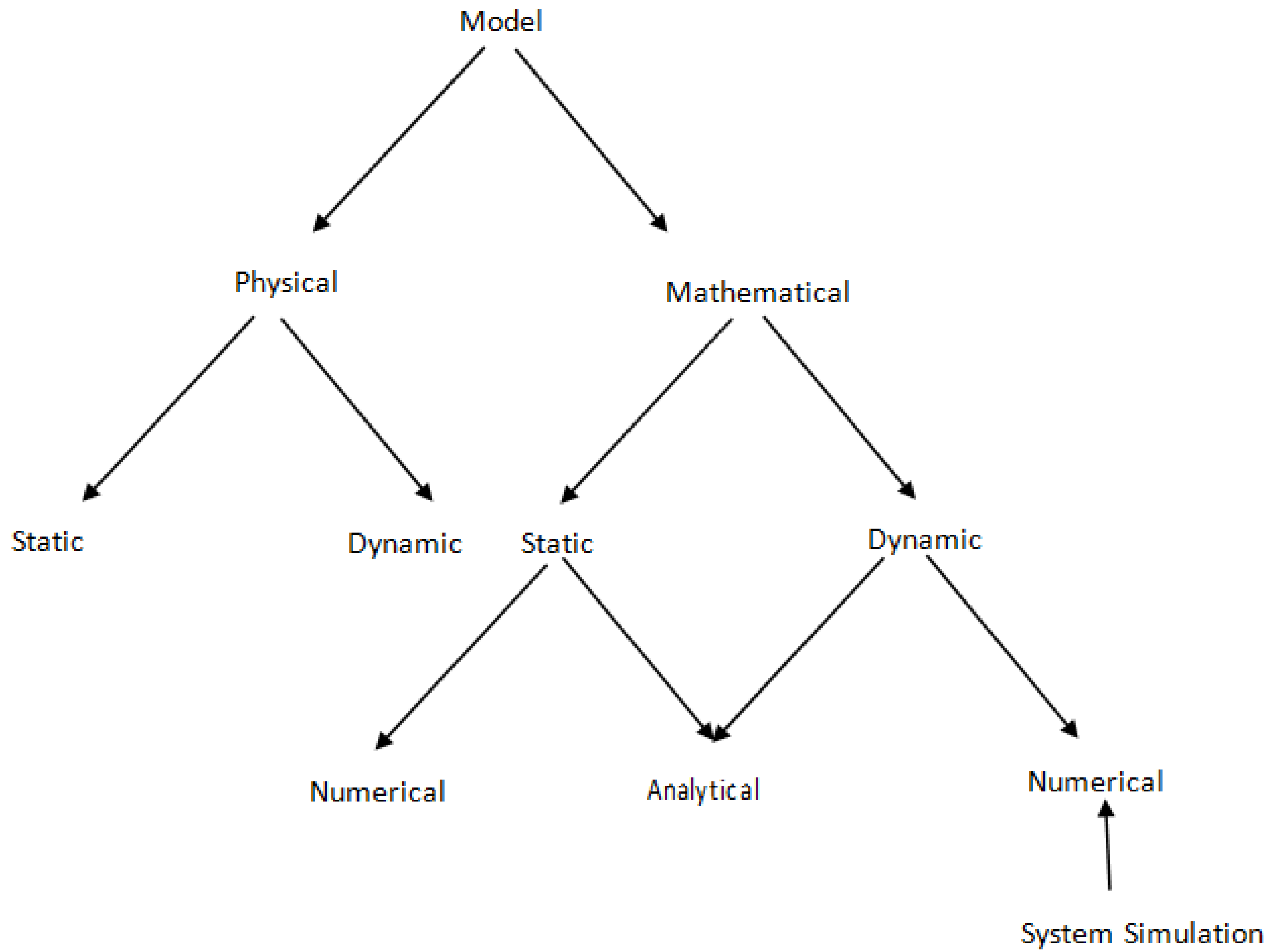
3. Accuracy:

- The accuracy of the information gathered for the model should be considered.

4. Aggregation:

- A further factor to be considered as the extent to which the number of individual entities can be grouped together into larger entities.

Types of system model:



Types of system model:

- Physical models ***are based on some analogy between systems*** like mechanical and electrical. The system attributes are represented by such measurements like voltage.
- Mathematical models ***use symbolic notation and mathematical equations to represent the system attributes***. The attributes represented by variables and activities by mathematical functions that inter-relate the variables.
- Static models ***can only show the values that system attributes take when the system is in balance*** .Dynamic model , ***follow the changes over time that result from system activities.***
- Applying analytical techniques means using the deductive (**logical**) reasoning of mathematical theory to solve a model. For E.g. linear differential equation
- Numerical methods involves applying conceptual (**theoretical**) procedures to solve equations.

Types of system model:

1 . Static physical model

Represents a system at a particular point of time and also known as Monte- Carlo simulation.

In this model, the measurements are taken to represent attributes of the system being study under one set of equilibrium (balance) condition.

In this case, the measurement do not translate directly into system attribute values.

Well known laws of similitude are used to convert measurement on the scale model to the values that will occur in the real system.


MathWorks

1 . Static physical model …..

They are used in wind tunnels and water tanks in the course of designing aircraft and ships.

Scientists have used this model in which spheres represent atoms and rods or specially shaped sheet of metal connect the spheres to represent atomic bonds.

They are sometimes said to be ionic models, a term meaning “look alike”.

Sometimes a static physical model is used as a means of solving equation with particular boundary conditions.

1 . Static physical model …..

For E.g. the flow of heat and the distribution of electric charge through space can be related by common equation.

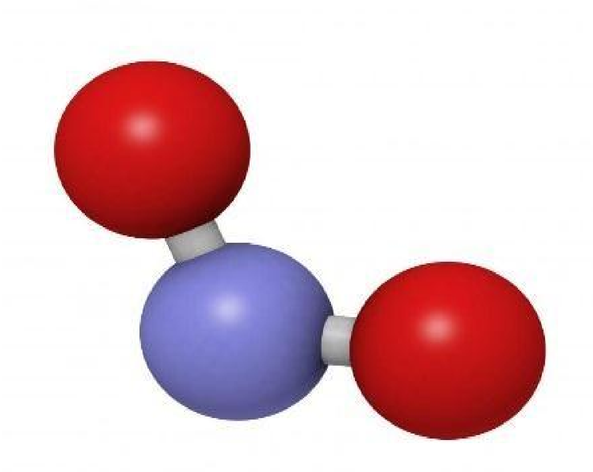


Fig: A stick model of water module

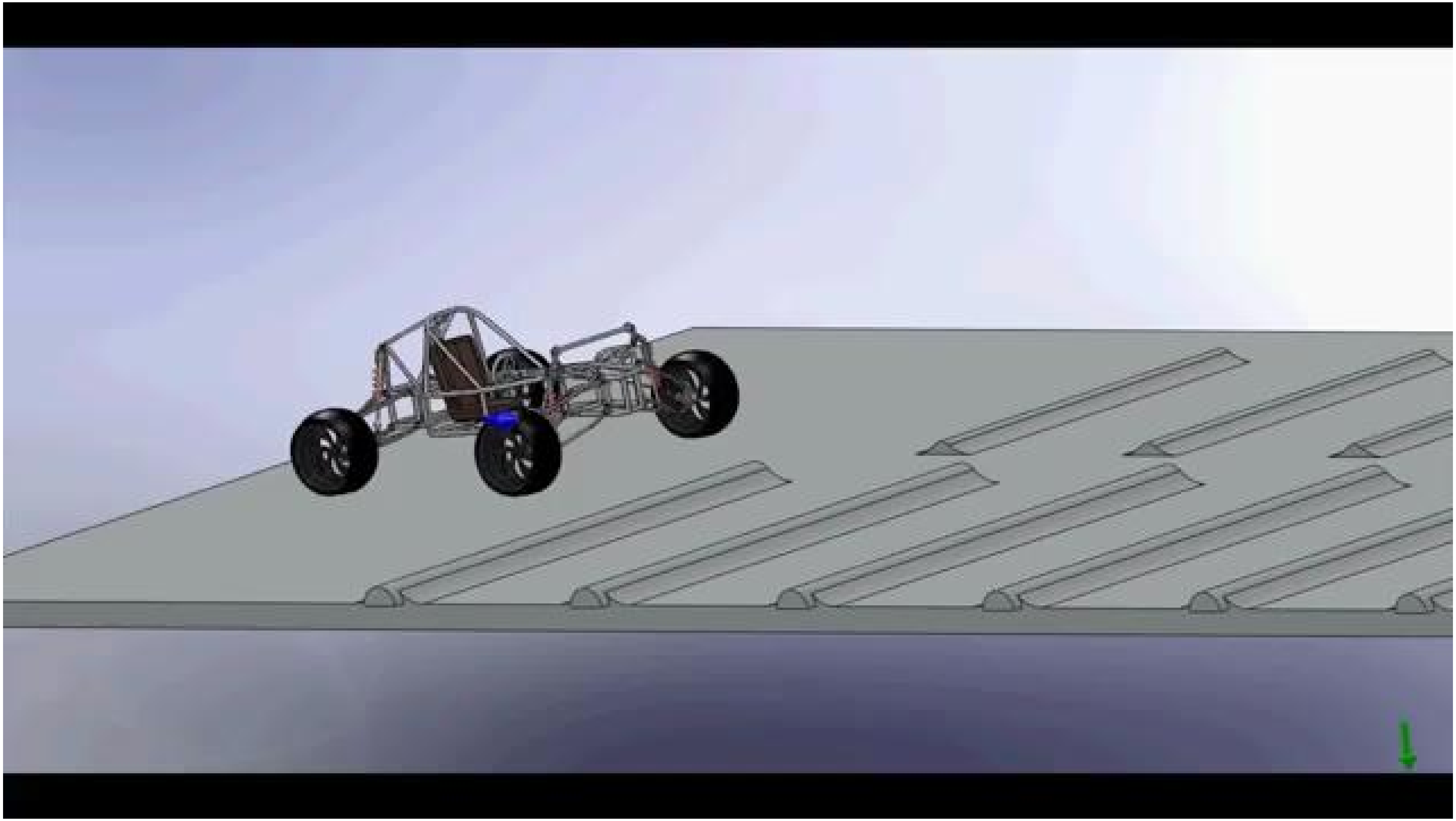
Types of system model:

2 . Dynamic physical model

Represents systems as they change over time. Ex: Simulation of a bank.

Dynamic physical models are ones which change with time or which are functions of time.

This model rely upon an analogy between the system being studied and some other system of different nature, the analogy usually depending upon an underlying similarities in the forces governing the behavior of the systems.



2 . Dynamic physical model…

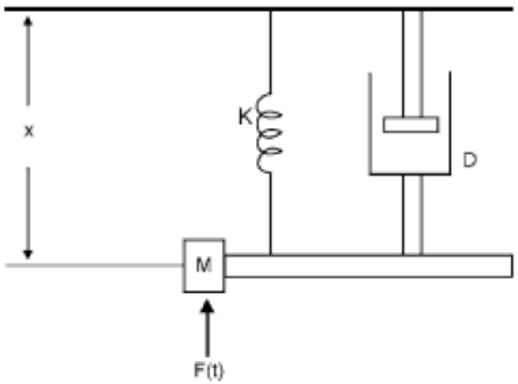


Fig. (a) Suspended weight attached with spring and piston of Mechanical System

Fig (a) represents a mass that is subjected to an applied force $F(t)$ varying with time, a spring whose force is proportional to its extension or contraction, and shock absorber (Damper) that exerts a damping force proportional to the velocity of the mass. It can be shown that the motion of the system is described by the differential equation

$$M\ddot{x} + D\dot{x} + kx = kF(t) \qquad \text{.....................} \; (i)$$

Where, x is distance moved

M is the mass,

K is the stiffness of spring,

D is the damping factor of Damper

2 . Dynamic physical model…

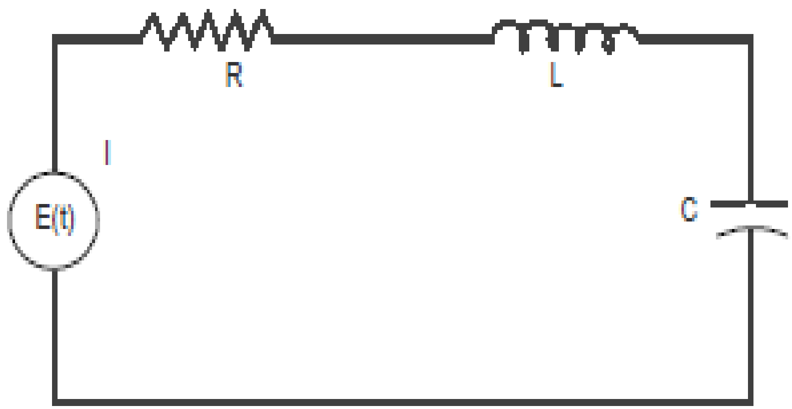


Fig. represents an electrical circuit with an inductance L, resistance R and capacitance C connected in series with a voltage source that varies in time according to function $E(t)$. If q is the charge on capacitor it can be shown that the barrier of the circuit is governed by following differential equation

2 . Dynamic physical model…

$$L\ddot{q} + R\dot{q} + \frac{1}{c}q = \frac{1}{c}E(t) \qquad \text{.....................} (ii)$$

From the above two equations (i) and (ii),

$Mechanical$	$Electrical$
x	q
$v(\dot{x})$	$i(\dot{q})$
$F(t)$	$E(t)$
M	L
D	R
K	$\frac{1}{c}$

Thus, same mathematical model, by using different constants can give the solution for both mechanical and electric circuit.

Types of system model:

3. Static mathematical model

A static model gives the relationship between the system attributes when the system is in equilibrium (balance).

If the point of equilibrium is change by alternating any of the attribute values, the model enables the new values for all the attributes to be derived.

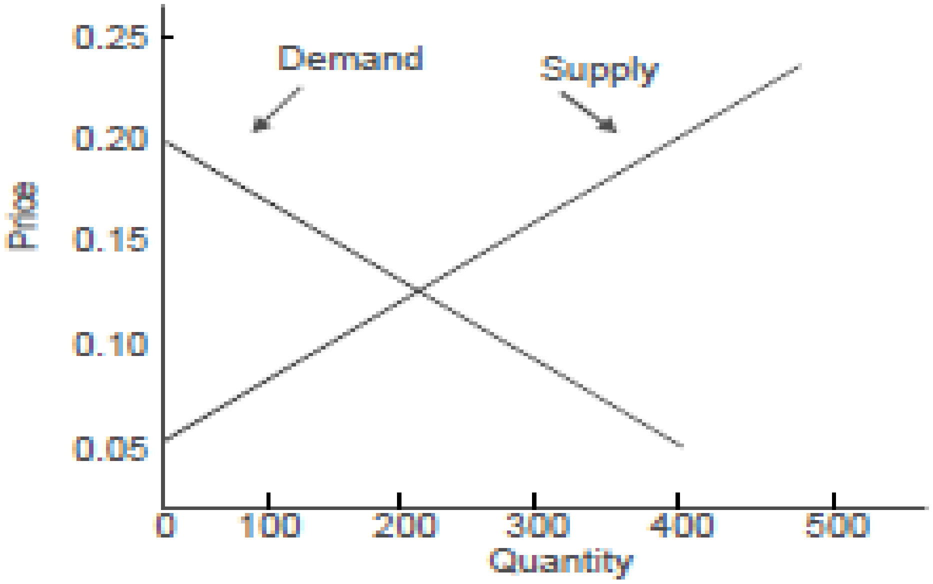
But doesn't show the way in which they changed to their new value.

If mathematical model doesn't involve time i.e. system does not change with time, it is called static mathematical model of the system.

3. Static mathematical model…

For e.g. we look at the case of static mathematical model from industry. In marketing a commodity generally there is a balance between supply and demand. Both factors depend on price. Demand for the commodity will be low when the price is high and it will increase as the price drops. If we take the simplistic linear case the relationship between demand (Q) and price (P) might be represented by the straight line.

3. Static mathematical model…



Fig(a): Linear Market Model

3. Static mathematical model…

Let us model this situation mathematically. If we denote price by P, supply by S and demand by D, and assuming the price equation to be linear we have

$D = a - bP$ where a= demand price

$S = c + dP$ c= supply price

$S = D$

3. Static mathematical model…

In the above equations, a, b, c, d are parameters computed based on previous market data. Equation $S = D$ says supply should be equal to demand so that market price should accordingly be adjusted. Let us take values of $a = 500$, $b = 2000$, $c = -50$ and $d = 1500$. Value of c is taken negative, since supply cannot be possible if price of the item is zero. In this case no doubt equilibrium market price will be

$$P = \frac{a-c}{b+d} = \frac{550}{3500} = 0.1571$$

And, $S = 186$

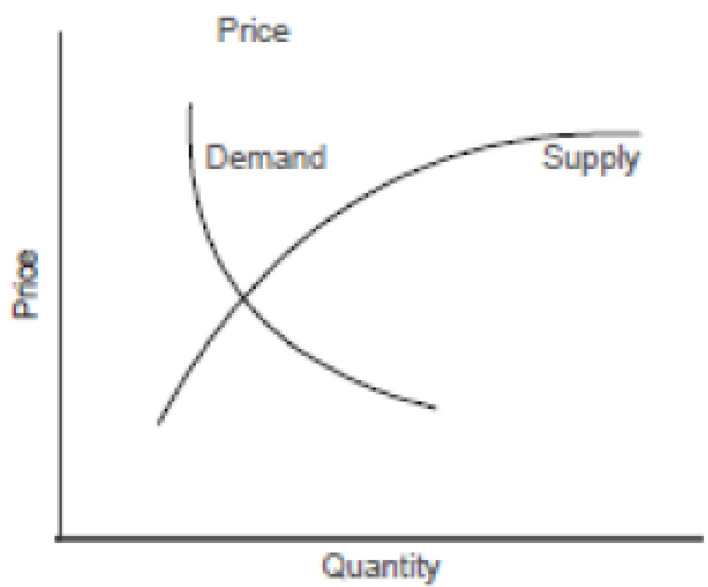


Fig: (b) Non Linear Market Model

3. Static mathematical model…

In the first model we have taken simple linear case, but fig (b) might be complex.

In that case the solution may not be so simple. More usually, the demand and supply are depreciated by curves with slopes upward and downward respectively.

It may not be possible to express the relationships by equations that can be solved easily.

Some numerical or graphical methods are used to solve such relations. In addition, it is difficult to get the values of the coefficients of the model. Observations over the extended period of time, however, will establish the slopes (that is values of b and d) in the neighborhood of the equilibrium points.

These values will often fluctuate under the global and local economic conditions.

Types of system model:

4. Dynamic mathematical model

Dynamic mathematical model allows the changes of system attributes to be derived as a function of time.

The derivation may be made with an analytical solution or with a numerical computation, depending upon the complexity of the model.

The equation to describe the behavior of the car wheel is an example of a dynamic mathematical model.

It is not possible to find analytic solution of this equation and one has to adopt the numerical methods.

4. Dynamic mathematical model…

We divide equation by M and write in the following form

$$\ddot{x} + 2\zeta w\dot{x} + w^2 x = w^2 F(t)$$

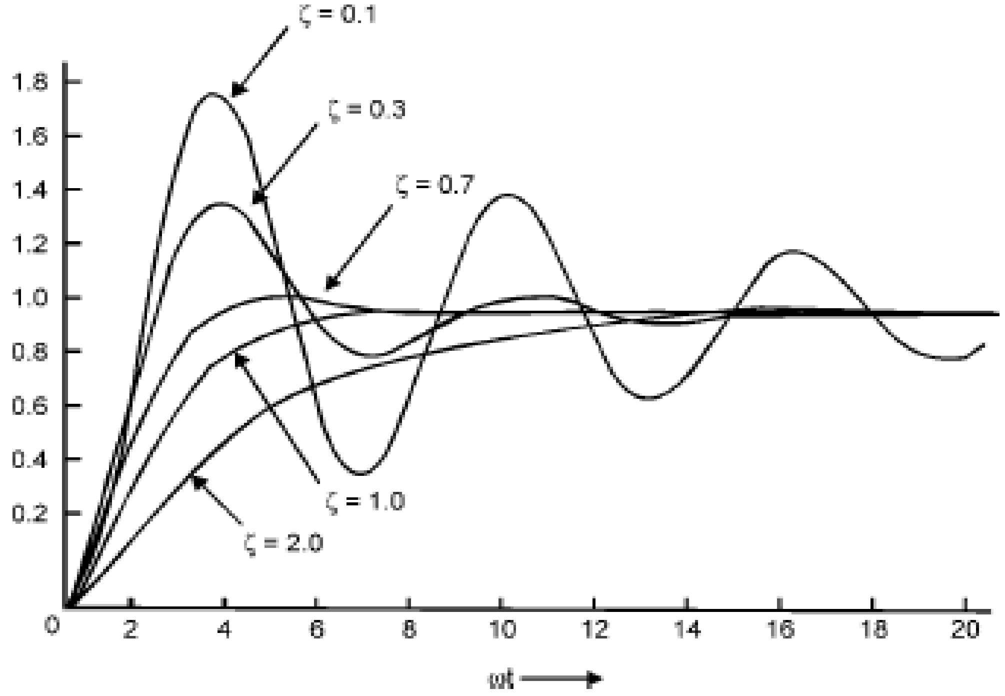
Where, $2\zeta w = \frac{D}{M}$ and $w^2 = \frac{k}{M}$

The solution can be given in terms of the variable$w(t)$, where w is the oscillation given by,

$$w = 2\pi f$$

And f is the number of cycles per second

4. Dynamic mathematical model…



The fig. shows how x varies in response to a steady force applied at time t =0. It can be seen that when $\zeta < 1$, the motion is oscillatory.

Distributed lag model

- ***Models that have the property of changing only at fixed interval of time.***
- It is used to predict current values of a dependent variable based on both the current values of an explanatory variable (independent variable) and the lagged (past period) values of this explanatory (helpful) variable.
- In economic studies some economic data are collected over uniform time interval such as a month or year. This model consists of linear algebraic equations that represent continuous system but data are available at fixed points in time.

Distributed lag model

- ***Any variable that can be expressed in the form of its current value and one or more previous value is called lagging variable.*** And hence this model is given the name distributed lag model.
- The variable in a previous interval is denoted by attaching –n suffix to the variable. Where –n indicate the nth interval.

Advantages of distributed lag model

- Simple to understand and can be computed by hand, computers are extensively used to run them.
- There is no need for special programming language to organize simulation task.

Steps in a Simulation study

1. Problem formulation (Invention)

Every study begins with a *statement of the problem*, provided by policy makers.

Analyst ensures its clearly understood. If it is developed by analyst policy makers should understand and agree with it.

Steps in a Simulation study

2. Setting of objectives and overall project plan

The objectives indicate the questions to be answered by simulation.

At this point a determination should be made concerning whether simulation is the appropriate methodology.

Assuming it is appropriate, the overall project plan should include

- A statement of the ***alternative systems***
- A method for evaluating the effectiveness of these alternatives
- Plans for the study in terms of the number of people involved
- Cost of the study
- The number of days required to accomplish each phase of the work with the anticipated (predicted) results.

Steps in a Simulation study

3. Model conceptualization

The construction of a model of a system is probably as much art as science.

The art of modeling is enhanced by an ability

- To abstract the essential features of a problem
- To select and modify basic assumptions that characterize the system
- To enrich and elaborate the model until a useful approximation results

Thus, it is best to start with a simple model and build toward greater complexity. Model conceptualization enhance the quality of the resulting model and increase the confidence of the model user in the application of the model.

Steps in a Simulation study

4. Data collection

There is a constant interplay between the construction of model and the collection of needed input data. Done in the early stages.

Objective kind of data are to be collected.

Steps in a Simulation study

5. Model translation

Real-world systems result in models that require a great deal of information storage and computation.

It can be programmed by using simulation languages or special purpose simulation software.

Simulation languages are powerful and flexible. Simulation software models development time can be reduced.

Steps in a Simulation study

6. Verified

It pertains to be computer program and checking the performance.

If the input parameters and logical structure are correctly represented, verification is completed.

Steps in a Simulation study

7. Validated

It is the determination that a model is an accurate representation of the real system.

Achieved through calibration of the model, an iterative process of comparing the model to actual system behavior and the discrepancies between the two.

Steps in a Simulation study

8. Experimental Design

The alternatives that are to be simulated must be determined. Which alternatives to simulate may be a function of runs.

For each system design, decisions need to be made concerning

- Length of the initialization period
- Length of simulation runs
- Number of replication to be made of each run

Steps in a Simulation study

9. Production runs and analysis

They are used to estimate measures of performance for the system designs that are being simulated.

Steps in a Simulation study

10. More runs

Based on the analysis of runs that have been completed.

The analyst determines if additional runs are needed and what design those additional experiments should follow.

Steps in a Simulation study

11. Documentation and reporting

Two types of documentation.

- Program documentation
- Process documentation

Program documentation

Can be used again by the same or different analysts to understand how the program operates. ***Further modification will be easier.*** Model users can change the input parameters for better performance.

Process documentation

Gives the history of a simulation project. The result of all analysis should be reported clearly and concisely in a final report. This enable to review the final formulation and alternatives, results of the experiments and the recommended solution to the problem. The final report provides a vehicle of certification.

Steps in a Simulation study

12. Implementation

Success depends on the previous steps. If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.

Steps in a Simulation study

1. Problem formulation
2. Setting of objectives and overall project plan
3. Model conceptualization
4. Data collection
5. Model translation
6. Verified
7. Validated
8. Experimental Design
9. Production runs and analysis
10. More runs
11. Documentation and reporting
12. Implementation

The simulation model building can be broken into 4 phases:

I Phase

- Consists of steps 1 and 2
- It is period of discovery/orientation
- The analyst may have to restart the process if it is not fine-tuned
- Recalibrations and clarifications may occur in this phase or another phase.

II Phase

- Consists of steps 3,4,5,6 and 7
- A continuing interplay is required among the steps
- Exclusion of model user results in implications during implementation

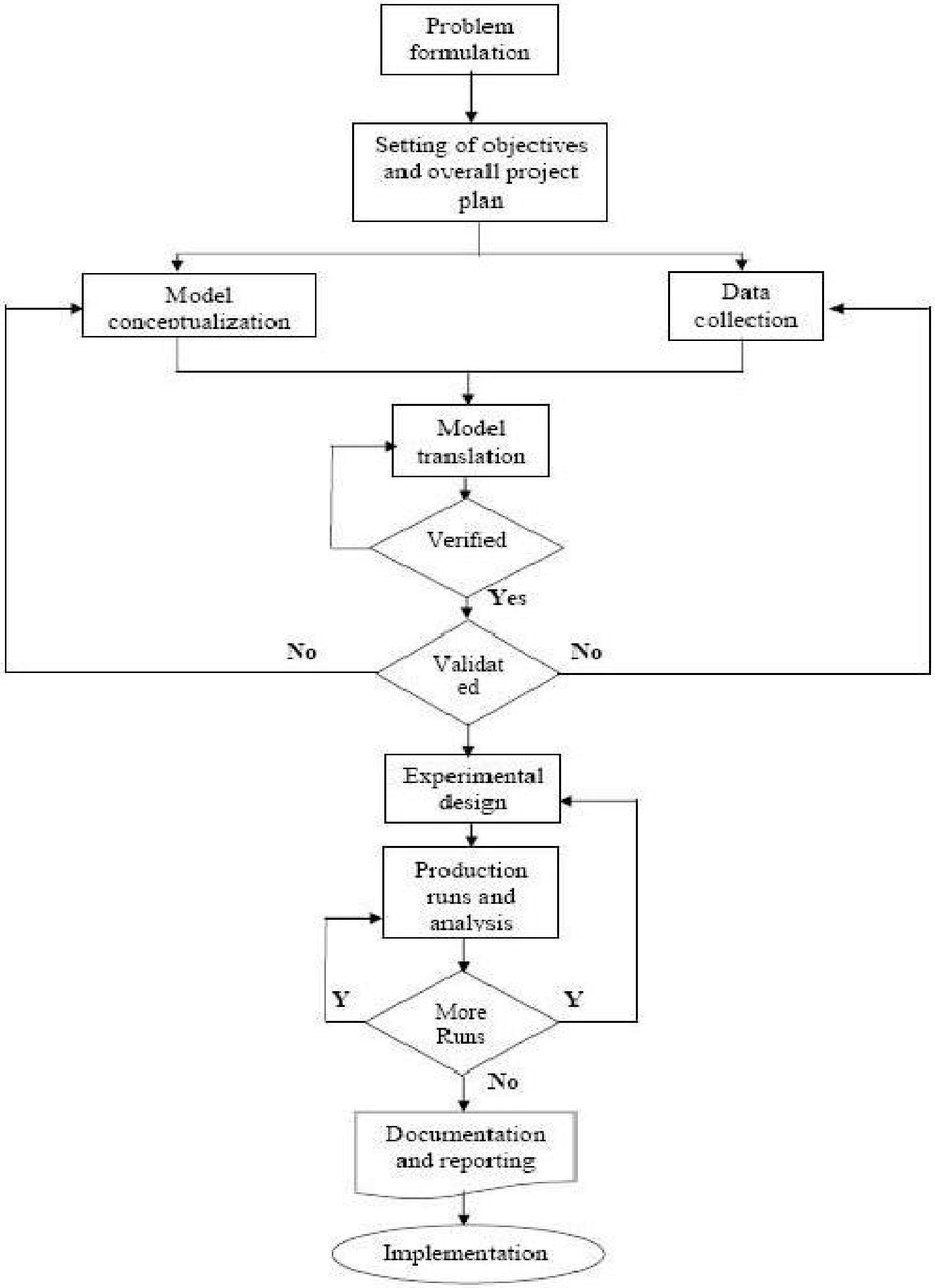
The simulation model building can be broken into 4 phases:

III Phase

- Consists of steps 8,9 and 10
- Conceives a thorough plan for experimenting
- Discrete-event stochastic is a statistical experiment
- The output variables are estimates that contain random error and therefore proper statistical analysis is required.

IV Phase

- Consists of steps 11 and 12
- Successful implementation depends on the involvement of user and every steps successful completion.


Problem formulation
Setting of objectives and overall project plan
Model conceptualization
Data collection
Model translation
Verified
Yes
No
Validat ed
No
Experimental design
Production runs and analysis
Y
More Runs
Y
No
Documentation and reporting
Implementation

Advantages of simulation

- Simulation can also be used to study systems in the design stage.
- Simulation models are run rather than solver.
- New policies, operating procedures, decision rules, information flows, organizational procedures, and so on can be explored without disrupting ongoing operations of the real system.
- New hardware designs, physical layouts, transportation systems, and so on can be tested without committing resources for their acquisition.

Advantages of simulation

- **Hypotheses** about how or why certain phenomena occur can be tested for feasibility
- Time can be compressed or expanded to allow for a speed-up or slow-down of the phenomena under investigation.
- Insight can be obtained about the interaction of variables.
- Insight can be obtained about the importance of variables to the performance of the system.

Advantages of simulation

- **Bottleneck** analysis can be performed to discover where work in process, information, materials, and so on are being delayed excessively.
- A simulation study can help in understanding how the system operates rather than how individuals think the system operates.
- what-if" questions can be answered. Useful in the design of new systems.

Limitations/Disadvantages of the Simulation technique

- **Model building requires special training.**
  - It is an art that is learned over time and through experience.
- **Simulation results can be difficult to interpret**
  - Most simulation outputs are essentially **random variables** (they are usually based on random inputs), so it can be hard to distinguish whether an observation is a result of system interrelationships. of randomness.

Limitations/Disadvantages of the Simulation technique

- **Simulation modeling and analysis can be time consuming and expensive.**
  - Skimping (Economical) on resources for modeling and analysis could result in a simulation model or analysis that is not sufficient to the task.
- **Simulation is used in some cases when an analytical solution is possible, or even preferable.**
  - This might be particularly true in the simulation of some waiting lines where closed-form queuing models are available.

When simulation is the appropriate tool:

- Simulation enables the study of and experimentation with the internal interactions of a complex system, or of a subsystem within a complex system.
- Informational, organizational and environmental changes can be simulated and the effect of those alternations on the model's behavior can be observer.
- The knowledge gained in designing a simulation model can be of great value to suggesting improvement in the system under investigation.
- By changing simulation inputs and observing the resulting outputs, valuable insight may be obtained into which variables are most important and how variables interact.

When simulation is the appropriate tool:

- Simulation can be used as a pedagogical(academic) device to reinforce (highlight) analytic solution methodologies.
- Simulation can be used to experiment with new designs or policies prior implementation, so as to prepare for what may happen.
- Simulation can be used to verify analytic solution.
- By simulating different capabilities for a machine, requirements can be determined.
- Simulation models designed for training, allow learning without the cost and disruption of on-the-job learning.
- Animation shows a system in simulated operation so that the plan can be visualized.
- The modern system (factory, water fabrication plant, service organization etc.) is so complex that the interactions can be treated only through simulation.

When Simulation Is Not Appropriate

- Simulation should not be used when the problem can be solved using common sense.
- *Simulation should not be used if the problem can be solved analytically (logically).*
- Simulation should not be used if it is easier to perform direct experiments.
- Do not to use simulation, if the costs exceed the savings
- Simulation should not be performed if the resources or time are not available. if a decision in needed is two weeks and a simulation will take a month, the simulation study is not advised

When Simulation Is Not Appropriate

- **Simulation takes data, sometimes lots of data. If no data is available, not even estimates, simulation is not advised**
- If there is not enough time or the personnel are not available, simulation is not appropriate.
- If managers have unreasonable expectations say, too much too soon— or the power of simulation is overestimated, simulation may not be appropriate
- If system behavior is too complex or can't be defined, simulation is not appropriate.

Applications:

- Designing and analyzing manufacturing systems
- Evaluating (Calculating) H/W and S/W requirements for a computer system
- Evaluating a new military weapons system or tactics
- Determining ordering policies for an inventory system
- Designing communications systems and message protocols for them
- Designing and operating transportation facilities such as freeways, airports, subways, or ports
- Evaluating designs for service organizations such as hospitals, post offices, or fast-food restaurants
- Analyzing financial or economic systems

Applications:

Manufacturing Applications

- Analysis of electronics assembly operations
- Design and evaluation of a selective assembly station for high-precision scroll compressor shells
- Comparison of dispatching rules for semiconductor manufacturing using large-facility models
- Evaluation of cluster tool throughput for thin-film head production
- Determining optimal lot size for a semiconductor back-end factory
- Optimization of cycle time and utilization in semiconductor test manufacturing
- Analysis of storage and retrieval strategies in a warehouse
- Investigation of dynamics in a service-oriented supply chain
- Model for an Army chemical munitions disposal facility

Applications:

Semiconductor Manufacturing

- Comparison of dispatching rules using large-facility models
- The corrupting influence of variability
- A new lot-release rule for wafer fibs
- Assessment of potential gains in productivity due to proactive reticle management
- Comparison of a 200-mm and 300-mm X-ray lithography cell
- Capacity planning with time constraints between operations
- 300-mm logistic system risk reduction

Applications:

Construction Engineering

- Construction of a dam embankment
- Trenchless renewal of underground urban infrastructures
- Activity scheduling in a dynamic, multi project setting
- Investigation of the structural steel erection process
- Special-purpose template for utility tunnel construction

Applications:

Military Application

- Modeling leadership effects and recruit type in an Army recruiting station
- Design and test of an intelligent controller for autonomous underwater vehicles
- Modeling military requirements for non war fighting operations
- Multi trajectory performance for varying scenario sizes
- Using adaptive agent in U.S Air Force pilot retention

Applications:

Logistics, Transportation, and Distribution Applications

- Evaluating the potential benefits of a rail-traffic planning algorithm
- Evaluating strategies to improve railroad performance
- Parametric modeling in rail-capacity planning
- Analysis of passenger flows in an airport terminal
- Proactive flight-schedule evaluation
- Logistics issues in autonomous food production systems for extended- duration space exploration
- Sizing industrial rail-car fleets
- Product distribution in the newspaper industry
- Design of a toll plaza
- Choosing between rental-car locations
- Quick-response replenishment

Applications:

Business Process Simulation

- Impact of connection bank redesign on airport gate assignment
- Product development program planning
- Reconciliation of business and systems modeling
- Personnel forecasting and strategic workforce planning

Applications:

Human Systems

- Modeling human performance in complex systems
- Studying the human element in air traffic control

Differential and partial differential equatio

Continuous model

- **When continuous system is modeled mathematically, the variables of model representing the attribute of system are controlled by continuous functions.**
- The distributed lag model is an example of a continuous model. Since in continuous system, the relationship between variables describe the rate at which the value of variable change, these system consist of differential equations.
- **Continuous system simulation uses the notation of differential equation to represent the change on the basic parameter of the system with respect to time.**
- Hence the Mathematical model for continuous system simulation is usually represented by differential and partial differential equations.

Differential and partial differential equation

Differential Equations

- ***A differential equation is a mathematical equation that relates some function with its derivatives.***
- An example of a linear differential equation with constant coefficients to describe the **wheel suspension system of an automobile** can be given as

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

The dependent variable x appears with its first and second order derivative $\dot{x}$ & $\ddot{x}$ and the term involving these quantities are multiplied by constant coefficient and added. The quantity $F(t)$ is an input to the system depending upon the independent variable t.

Differential and partial differential equatio

- **A dependent variable is a variable whose value depends upon independent variables.**
- **The dependent variable is what is being measured in an experiment or evaluated in a mathematical equation. The dependent variable is sometimes called "*the outcome variable*.“ In a simple mathematical equation, for example:**

$$a = b/c$$

the dependent variable, *a* , is determined by the values of *b* and *c* .

Differential and partial differential equation:

- If the dependent variable or any of its derivate appears in any other form such as being raised to a power or are combined in any other way for e.g. by being multiplied together, the differential equation is said to be **non-linear**.
- **When more than one independent variable occurs in a differential equation, the differential equation is said to be partial differential equation.**
- It involves the derivative of the same dependent variable with respect to each of the independent variable.

Differential and partial differential equatio

- ***A linear equation is always a polynomial of degree 1 (for example x+2y+3=0).***
- ***Every other equation is nonlinear. Higher degree polynomials are nonlinear (x^2+3x+2=0).***

E.g. Equation of heat flowing through 3-D body i.e.

$$\frac{\partial u}{\partial t} - \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 0$$

Here the dependent variable is u and independent variable is x, y, z and t.

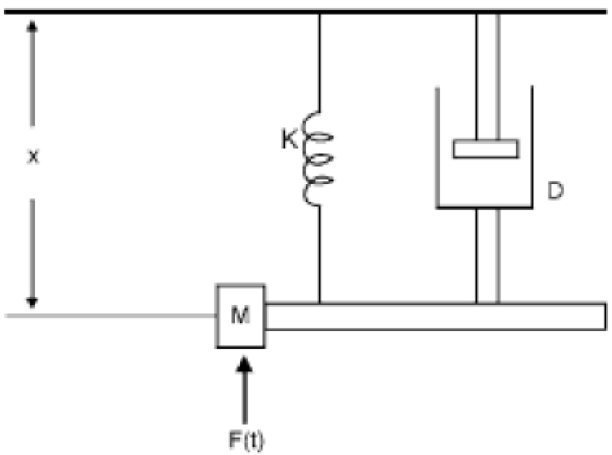


Fig. (a) Suspended weight attached with spring and piston of Mechanical System

Fig (a) represents a mass that is subjected to an applied force $F(t)$ varying with time, a spring whose force is proportional to its extension or contraction, and shock absorber (Damper) that exerts a damping force proportional to the velocity of the mass. It can be shown that the motion of the system is described by the differential equation

$$M\ddot{x} + D\dot{x} + kx = kF(t) \quad \text{.....................} \quad (i)$$

Where, x is distance moved

M is the mass,

K is the stiffness of spring,

D is the damping factor of Damper

Differential and partial differential equatio

- Differential equation, both linear and non-linear occurs repeatedly in scientific and engineering studies. It shows the rate of change of physical and chemical process in term of mathematical equations. Differential coefficient can also represent growth rate.
- Let us illustrate how differential equation can represent engineering problems. We will show how the equation describing the automobile wheel suspension system is derived from the mechanical principle.

$$\dot{x}\ represent\ velocity$$

$$\ddot{x}\ represents\ acceleration$$

Differential and partial differential equation

- In absence of other process $M\ddot{x} = KF(t)$. However the shock absorber exerts a resisting force that depend on the velocity and increases as velocity rises, it is represented by $D\dot{x}$. Similarly the spring exert a resistance force which depends upon the extent to which it has been compressed, and represented by Kx. Since both of these forces opposes the motion they are subtracted from the applied force to give the equation of motion.

$$M\ddot{x} = KF(t) - D\dot{x} - Kx$$

- It is a linear differential equation with constant coefficient representing the equation for suspension of automobile wheel as mentioned before. It can be also solved analytically.

CHAPTER 1

Finished !!!