MARKOV CHAIN

UNIT 4

Markov chain

- **A Markov chain is a discrete-time process for which the future behavior, given the past and the present, only depends on the present and not on the past.**
- A Markov process is the continuous-time version of a Markov chain.
- A Markov chain is a mathematical model for a process which moves step by step through various state.
- The probability that the process moves from any given state to any other particular state is always same regardless of the history of the process.

Markov chain

- A Markov chain consists of state and transition probabilities.
- ***Each transition probabilities is the probability of moving from one state to another in one step.***
- The transition probabilities are independent of the past and depend only on the two states involved.
- ***The matrix of transition call probabilities are called transition matrix.***
- Markov modeling is an extremely important to the field of modeling and analysis of telecommunication networks.

Key features of **Markov chain**:

A sequence of trail of an experiment is a Markov chain if:

1. The outcome of each experiment is one of a set of discrete state.
2. The outcome of the experiment depends only on the present state and not on the past state.
3. The transition probability remains constant from one to the next.

Markov chain

Property:

- The state of the system at time $t + 1$ depends only on the state of the system at time t.

$$P\left[X_{t+1} = b \mid X_t = a\right] = P_{ab}$$

Markov chain

Examples 1:

1. **Weather problem**

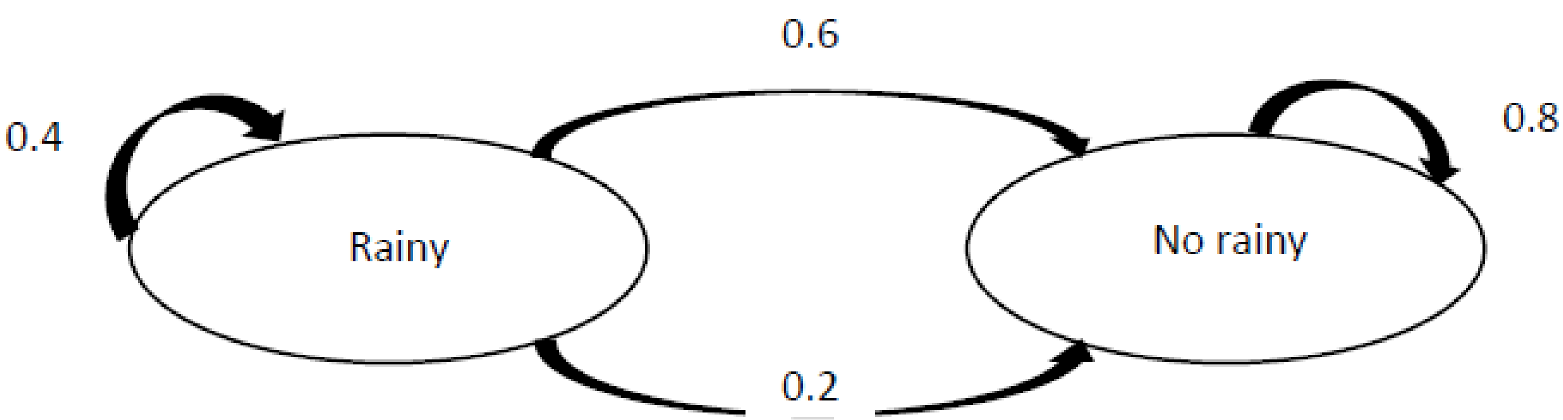
Rainy today	=> 40% Rainy tomorrow
	=> 60% not Rainy tomorrow
Not rainy today	=> 20% Rainy tomorrow
	=> 80% not Rainy tomorrow

What will be probability if todays is not raining then not rain the day after tomorrow?

Markov chain

Solution :

Markov chain diagram:



Markov chain

Solution :

P (not rainy - ________- not rainy)

= P (not rainy - not rainy - not rainy) + P (not rainy - rainy - not rainy)

= 0.8 * 0.8 + 0.2 * 0.6

= 0.64 + 0.12

=0.76

Markov chain

Transition matrix

$$P = \begin{array}{r c} \text{Rainy} \rightarrow & \multirow{2}{*}{$\begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$} \\ \text{Not rainy} \rightarrow & \\ & \begin{matrix} \uparrow & \uparrow \\ \text{Rainy} & \text{Not rainy} \end{matrix} \end{array}$$

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Transition matrix

$$P^2 = \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix} \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.16 + 0.12 & 0.24 + 0.48 \\ 0.08 + 0.16 & 0.12 + 0.64 \end{pmatrix}$$

$$= \begin{pmatrix} 0.28 & 0.72 \\ 0.24 & 0.76 \end{pmatrix}$$

Thus, the probability of not rainy the day after tomorrow is 0.76.

Q> What will be probability if todays is not raining then rain the day after tomorrow?

Rainy today	=> 40% Rainy tomorrow
	=> 60% not Rainy tomorrow
Not rainy today	=> 20% Rainy tomorrow
	=> 80% not Rainy tomorrow

Solution :

= P(NR_R)

= P(NR-NR-R) + P(NR-R-R)

= 0.8 x 0.2 + 0.2 x 0.4

= 0.16 + 0.08

= 0.24

OR

Transition matrix

$$P^2 = \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix} \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.16 + 0.12 & 0.24 + 0.48 \\ 0.08 + 0.16 & 0.12 + 0.64 \end{pmatrix}$$

$$= \begin{pmatrix} 0.28 & 0.72 \\ 0.24 & 0.76 \end{pmatrix}$$

Q> What will be probability if todays is not raining then rain after three days?

Rainy today	=> 40% Rainy tomorrow => 60% not Rainy tomorrow
Not rainy today	=> 20% Rainy tomorrow => 80% not Rainy tomorrow

Solution :

= P(NR - _______ - _______ - R)

= P(NR-NR-NR-R) + P(NR-NR-R-R) + P(NR-R-NR-R) + P(NR-R-R-R)

= 0.8 x 0.8 x 0.2+ 0.8 x 0.2 x 0.4 + 0.2 x 0.6 x 0.2 + 0.2 x 0.4 x 0.4

= 0.128 + 0.064 + 0.024 + 0.032

= 0.248

OR

$$P^3 = \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix} . \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix} . \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.256 & 0.744 \\ 0.248 & 0.752 \end{pmatrix}$$

Markov chain

Example 2

2. **Coke - Pepsi**

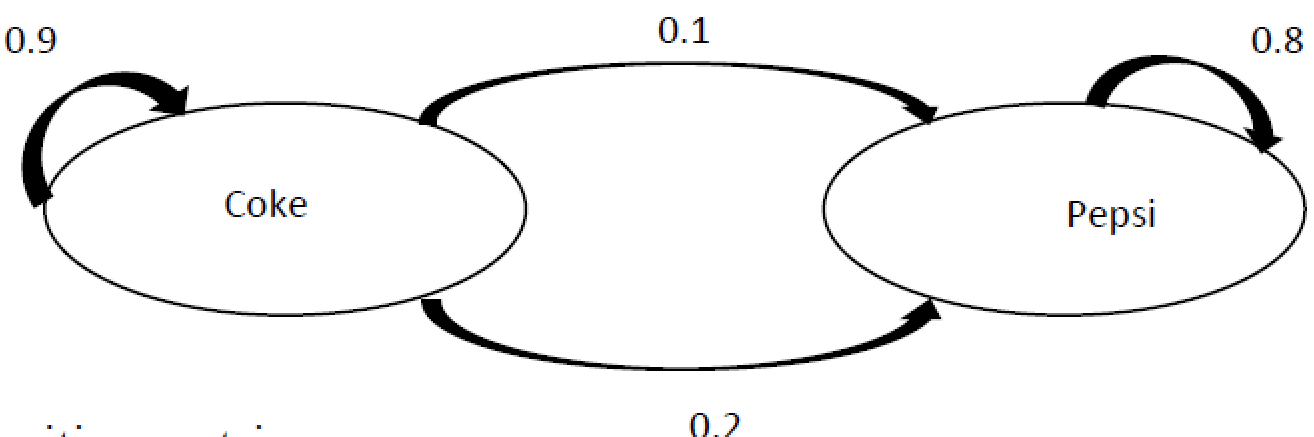
Coke => 90% Coke

Pepsi => 20% Coke

Markov chain

Solution :

Markov chain diagram:



Transition matrix:

$$P = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

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Q.N.> Given a person is currently a Pepsi purchaser. What is the probability of purchase of coke after two purchases from now?

Solution :

P (Pepsi _Coke)
= P (Pepsi - Pepsi - Coke) + P (Pepsi - Coke - Coke)
= 0.8 * 0.2 + 0.2 * 0.9
= 0.16 + 0.18
= 0.34

$$P = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

OR
Matrix Form

$$P^2 = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix} \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.91 + 0.02 & 0.09 + 0.08 \\ 0.18 + 0.16 & 0.02 + 0.64 \end{pmatrix}$$

$$= \begin{pmatrix} 0.83 & 0.17 \\ 0.34 & 0.66 \end{pmatrix}$$

Thus, the probability purchase of coke after two purchases from now is 0.34.

Markov chain

Q.N.> Given a person is currently a Coke purchaser. What is the probability of purchase of Pepsi after 3 purchase?

Solution :

From transition matrix,

$$P = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.83 & 0.17 \\ 0.34 & 0.66 \end{pmatrix} \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.747 + 0.034 & 0.083 + 0.136 \\ 0.306 + 0.132 & 0.034 + 0.528 \end{pmatrix}$$

$$= \begin{pmatrix} 0.781 & 0.219 \\ 0.438 & 0.562 \end{pmatrix}$$

Thus, P (Coke - - Pepsi) = 0.219

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Q.N.> Suppose 60% of all people drink Coke and 40% Pepsi. What fraction of people will be drinking Coke 3 weeks from now?

Solution :

From transition matrix, we have

$$P^3 = \begin{pmatrix} 0.781 & 0.219 \\ 0.438 & 0.562 \end{pmatrix}$$

Probability of drinking Coke 3 weeks from now,

$$P_{coke} = 0.6 * 0.781 + 0.4 * 0.438$$

$$= 0.6438$$

$$= 64.38\%$$

And, $P_{Pepsi} = 35.62\%$

Markov chain

Application:

- Connection admission control
- Bandwidth allocation
- Routing
- Queuing and Scheduling

Markov chain

Application:

Physics

- Markovian systems appear extensively in thermodynamics and statistical mechanics, whenever ***probabilities are used to represent unknown or unmodelled details of the system***, if it can be assumed that the dynamics are time-invariant, and that no relevant history need be considered which is not already included in the state description.

Markov chain

Application:

Queuing theory

- Markov chains are the ***basis for the analytical treatment*** of queues (queuing theory). Agner Krarup Erlang initiated the subject in 1917. This makes them critical for optimizing the performance of telecommunications networks, where ***messages must often compete for limited resources (such as bandwidth).***

Markov chain

Application:

Internet applications

- The ***Page Rank of a webpage*** as used by Google is defined by a Markov chain. It is the probability to be at page i in the stationary distribution on the following Markov chain on all (known) web pages

Markov chain

Application:

Statistics

- Markov chain methods have also become very important for ***generating sequences of random numbers*** to accurately reflect very complicated desired probability distributions, via a process called Markov chain Monte Carlo (MCMC) And many more.

Q.N.1> Define and describe Markov chain in detail with the help of suitable examples. Also describe at least three of application of Markov chain. (*TU 2073 / 10 MARKS*)

CHAPTER 4

Finished !!!